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Corroboration of the New Method of Obtaining Dissipation in Wake Flow

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A two-dimensional far wake of a circular cylinder is investigated for the average turbulent kinetic energy equation. The dissipation term is obtained from the $\overline{u^2}$ spectra using a new method called the zero-length method. This method has previously been tested successfully in boundary-layer flow, fully developed pipe flow, and diffuser flow. The dissipation obtained through this method in the present study compares very well with the best values of dissipation available up to date in a similar flow. Additionally, the results of dissipation satisfy the closing term of the energy equation. It seems that this method of measurement of dissipation is universal in nature.

Nomenclature

A = constant of order 1
D = diameter of wake generating cylinder
d = diameter of hot wire
E_1, E_2 = outputs from hot-wire sets
$E_1(k_1)$ = one-dimensional energy spectrum
f_k = Kolmogorov frequency
$G(\eta)$ = Gaussian function used to fit mean velocities
h = similarity function
i, j = indices in Cartesian tensor notation
k = yaw correction factor for x wire
k_1 = one-dimensional wave number
L = mean-velocity defect half-width of wake
l = sensitive length of hot wire
l_o = integral length scale
p = atmospheric pressure
$\overline{q^2}$ = trace of Reynolds stress tensor
Re_D = Reynolds number based on freestream velocity and cylinder diameter
U = local mean velocity
U_o = centerline mean-velocity defect
U_∞ = freestream velocity
u = x component of instantaneous fluctuating velocity
v = y component of instantaneous fluctuating velocity
w = z component of instantaneous fluctuating velocity
x = distance downstream from center of wake generating cylinder
y = distance perpendicular to wake generating cylinder and freestream
z = distance parallel to wake generating cylinder
ε = dissipation rate
η = nondimensional distance y/L
ν = kinematic viscosity of air
ρ = air density

Superscript
$\overline{}$ = time average

I. Introduction

IN a turbulent fluid flow, the conversion of turbulent energy into heat is an important physical process characterized by the dissipation rate. If one writes an equation of turbulent kinetic energy, this term has a similar magnitude to that of production and, thus, high importance is attached to it. In the equation of turbulent kinetic energy, most of the terms can be obtained experimentally, though not the dissipation and pressure diffusion terms. However, if one of the latter terms is measured accurately, then the remainder term can be obtained by difference. Until now, there seems to be no hope of measuring the pressure diffusion term, and so attempts should be made toward determining the dissipation rate experimentally. Toward this end, Azad and Kassab[1] have introduced the zero-length method of measurement of dissipation. They have used this method successfully in boundary-layer flow, fully developed pipe flow, and diffuser flow. In the latter flow, they used the required high-intensity correction due to Lumley.[2] These flows represent partially or wholly wall bounded shear flows.

In the present investigation, it was decided to test this method for its validity in free shear flow. Incidentally, Browne et al.[3] have published a paper on wake flow in which they obtained dissipation by measuring 9 major terms out of the 12 terms that make up the total dissipation. This has been the most accurate measurement of dissipation in a wake to date, to our information. Hence, a wake flow similar to Browne et al.[3] was produced for the present experimentation. The purpose of this experimentation was to verify the zero-length method by first comparing it with the dissipation measurements of Browne et al.[3] and, second, from the balance of the kinetic energy equation.

In Sec. II, the experimental setup and procedure are described. Results and discussion are given in Sec. III, and finally, conclusions are presented in Sec. IV.

II. Experimental Setup and Procedure

The experiments were conducted in a closed-circuit, return-type wind tunnel. The test section was 0.76 m wide, 0.53 m high, and 1.83 m long. Uniformity and two dimensionality of the flow were checked at various locations in the test section before doing the experiments. The design of the test section ensured a zero pressure gradient along its length.

The experiments were carried out in a freestream velocity $U_\infty = 6.70$ m/s and the Reynolds number Re_D, based on the cylinder diameter and the freestream velocity, was 1160. The wake was generated by the technique of Cimbala,[4] who used a steel drill rod for that purpose. In the present experiments, a drill rod of diameter $D = 2.77$ mm was used. The rod was mounted horizontally in the central plane, 0.22 m from the start of the test section and normal to the flow.

The final measurements were done at $x/D = 420$, as in Browne et al.[3] This decision was based on the conclusion of self-preservation reached by Browne et al.[3] by analyzing the

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results of LaRue and Libby[5] and Antonia and Browne.[6] In addition, the present measurements of mean-velocity profiles at different locations after $x/D = 220$, as shown in Fig. 1, also indicated the location $x/D = 420$ to be in the self-preserving region. A brief summary of the experimental conditions for the present experiments and the experiments of Browne et al.[3] is given in Table 1.

The measurements were made using pitot and static pressure tubes and hot-wire anemometry. The single wire probes used were Dantec 55P05 gold plated boundary-layer-type probes, having a nominal length and diameter of 1.25 mm and 5 μm, respectively. The x-wire probes used were gold plated Dantec 55P51 type. Five Dantec 55P05 gold plated special boundary-layer-type probes were used for the spectra measurements. They were of different lengths and diameters, as given in Table 2.

The dimensions of these wires were measured using an ISI (Mini-SEM) scanning electron microscope. Only wires with $l/d \geq 200$ were used, as recommended by Azad and Kassab.[1] Disa 55M anemometers and Disa 55D10 linearizers were used for the hot-wire measurements. The anemometers were operated at an overheat ratio of 0.75. The measurements were obtained in analog form, using a TM377 (Tri-Met Instruments limited) multifunction turbulence processor, as well as in digital form, on a PDP 11/34 computer using a AD11-K A/D converter.

The mean-velocity field was measured with pitot and static pressure tubes as well as single and x-wire probes. The pressure measurements were obtained using a Combist differential micromanometer, sensitive up to 1/100 mm of water. All of

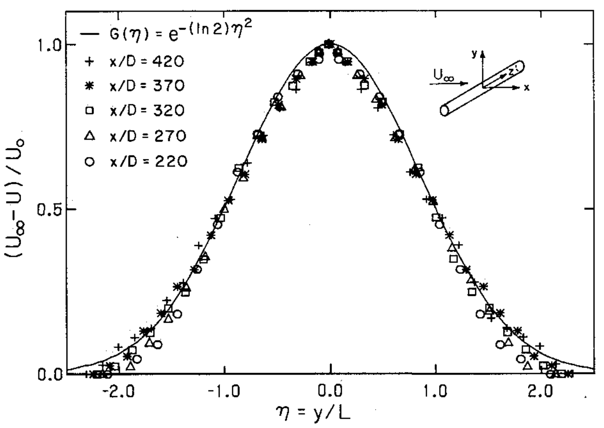

Fig. 1 Coordinate system and mean-velocity defect profiles in self-preserving coordinates.

Table 1 Experimental conditions

	Present experiments	Experiments of Browne et al.[3]
Freestream velocity, U_∞	6.70 m/s	6.70 m/s
Cylinder diameter, D	2.77 mm	2.67 mm
Location, x/D	420	420
Reynolds number based on D, Re_D	1160	1170
Mean-velocity defect, U_o	0.36 m/s	0.36 m/s
Mean-velocity defect half-width, L	13.03 mm	12.3 mm

Table 2 Dimensions of the wires for the spectra measurements

Number	Sensitive length, l, mm	Diameter, d, μm	l/d
1	0.49	2.42	202
2	1.04	3.10	335
3	1.23	5.41	227
4	1.68	4.40	382
5	1.98	4.60	430

the turbulent fluctuating quantities up to third-order moments were measured using single and x wires. It was decided to measure the triple velocity correlation $\overline{w^2v}$ (which was not measured by Browne et al.[3]) using the method adopted by Wygnanski and Fiedler[7] from Townsend,[8] which is outlined in the following.

The x wire was placed in the flow such that its plane was parallel to the flow and at an angle of 45 deg to the y and z axes. The outputs thus obtained from the hot-wire sets were

$$E_1 \propto u + k(v + w), \qquad E_2 \propto u - k(v + w)$$

therefore

$$E_1 - E_2 \propto v + w$$

Squaring this gives

$$\overline{(E_1 - E_2)^2} \propto \overline{v^2} + 2\,\overline{vw} + \overline{w^2}$$

and cubing gives

$$\overline{(E_1 - E_2)^3} \propto \overline{v^3} + 3\,\overline{v^2w} + 3\,\overline{vw^2} + \overline{w^3}$$

From symmetry about the y axis, $\overline{w^3}$, $\overline{vw}$, and $\overline{v^2w}$ vanish. Hence,

$$\overline{(E_1 - E_2)^2} \propto \overline{v^2} + \overline{w^2}$$

$$\overline{(E_1 - E_2)^3} \propto \overline{v^3} + 3\,\overline{vw^2}$$

which gives

$$\frac{\overline{(E_1 - E_2)^3}}{[\overline{(E_1 - E_2)^2}]^{3/2}} = \frac{\overline{v^3} + 3\,\overline{vw^2}}{(\overline{v^2} + \overline{w^2})^{3/2}}$$

Hence, $\overline{w^2v}$ was calculated from the above expression.

The u^2-spectra measurements were obtained with five different special probes as described earlier. The linearized signal was amplified and then low-pass filtered at the Kolmogorov frequency f_k using Krohn-Hite 3550 filters. The sensitivity of the linearized signal was 0.01 V = 0.01 m/s. The Kolmogorov frequency in the present flow was less than 2500 Hz and the signal was sampled at 25 KHz on the PDP 11/34. At this rate, the sampling time was about 1.5 min. The details of the procedure for spectra measurements are given in Azad and Kassab.[9]

The extent of the present measurements in the wake at $x/D = 420$ was from $y = -30$ to $+30$ mm. Since the wake was symmetrical, the spectra measurements were only made from $y = 0$ to $+30$ mm.

III. Results and Discussion

A. Mean Velocities

Profiles of mean velocity were obtained in the range $220 \leq x/D \leq 420$ and are plotted in the form $(U_\infty - U)/U_o$ vs $\eta = y/L$ in Fig. 1. All of the velocity profiles were seen to collapse to a single curve, within experimental error. The function defined by $G(\eta) = e^{-(\ln 2)\eta^2}$ for the mean-velocity profiles was introduced by Cimbala[4] and fitted all of his results very well. Our results also fit this function, as shown in Fig. 1, and this illustrates self-preservation of the wake in this range. Consequently, the variations of L and U_o obtained from the mean-velocity profiles show the following relationships:

$$L/D = 0.198[x/D + 144]^{1/2} \tag{1}$$

$$U_o/U_\infty = 1.28[x/D + 144]^{-1/2} \tag{2}$$

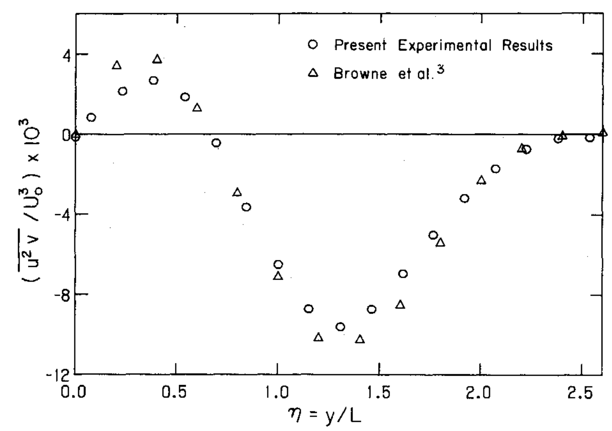

Fig. 2a Distributions of triple-velocity product $\overline{u^2v}$ across the wake.

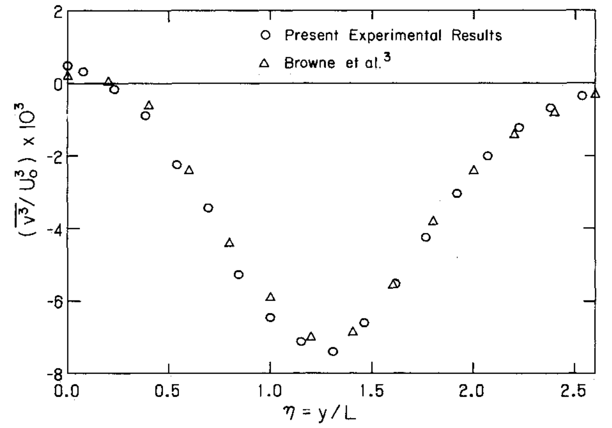

Fig. 2b Distributions of triple-velocity product $\overline{v^3}$ across the wake.

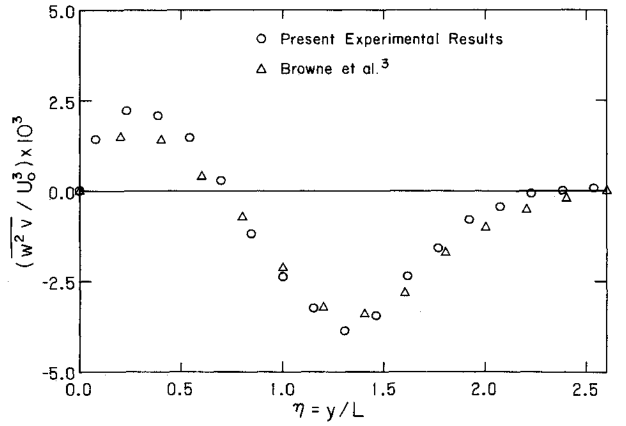

Fig. 2c Distributions of triple-velocity product $\overline{w^2v}$ across the wake.

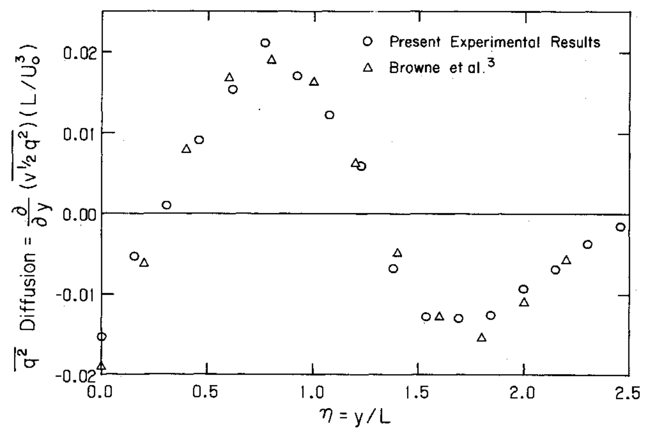

Fig. 3 Comparison of $\overline{q^2}$ diffusion with Browne et al.[3]

Similar relationships have also been obtained by Browne et al.,[3] which are

$$L/D = 0.2[x/D + 125]^{1/2}$$

$$U_o/U_\infty = 1.28[x/D + 125]^{-1/2}$$

Practically, only the constant in the parenthesis is different, i.e., in the present study it is 144 instead of 125 for Browne et al.[3] This difference could be due to the slight difference in the cylinder diameter. However, this constant does not influence the calculations significantly.

B. Energy Budget

Measurements of the different terms of the energy budget are systematically presented in this section. First of all, the three terms that were measured by conventional methods, namely, the $\overline{q^2}$ diffusion, the advection, and the production, are discussed. Then the dissipation term, which in the present study was measured using the zero-length method, is presented. Finally, all of these terms are given in the energy budget for the present study. The pressure diffusion term was obtained by the closure of the equation of turbulent kinetic energy.

1. $\overline{q^2}$ Diffusion

The transport of $\overline{q^2}$ across the wake is the most complicated phenomenon in the physics of turbulence of a wake. In a self-similar wake, it is given by the relation

$$\frac{\partial}{\partial y}\left(\frac{1}{2}\overline{vq^2}\right)$$

and it contains the three triple-velocity correlations $\overline{u^2v}$, $\overline{v^3}$, and $\overline{w^2v}$. These correlations are illustrated in the nondimensional form in Figs. 2a–2c. These figures also contain the points from the best fit curves of Browne et al.[3] and they show a good agreement with the present results, taking into consideration the scatter of the experimental results in Fig. 8 of Browne et al.[3] We wish to point out that Browne et al.[3] used measured values only for $\overline{u^2v}$ and $\overline{v^3}$ and scaled the $\overline{w^2v}$ correlation from the values of Fabris[10] using the relation

$$(\overline{w^2v})_{\text{Browne et al.}} = (\overline{u^2v})_{\text{Browne et al.}}\left(\frac{\overline{w^2v}}{\overline{u^2v}}\right)_{\text{Fabris}}$$

We have used the method introduced by Townsend[8] and later followed by Wygnanski and Fiedler[7] to obtain $\overline{w^2v}$. This method was outlined earlier in Sec. II.

The term $\frac{1}{2}\overline{vq^2}$ was obtained from the sum of the aforementioned three triple-velocity correlations. The lateral derivative of this term was obtained by fitting a quadratic curve through each of three consequentive points for numerical differentiation. Thus, the $\overline{q^2}$-diffusion term was obtained and is shown in nondimensional form in Fig. 3 together with the same term of Browne et al.[3] for comparison. Figure 3 shows that the present results compare very closely to the results of Browne et al.,[3] indicating a possibility of similar turbulent flow of the wake.

2. Advection

Measurements of $\overline{u^2}$, $\overline{v^2}$, and $\overline{w^2}$ were made with single and x-wire probes, and the advection term

$$U_\infty \frac{\partial}{\partial x}\left(\frac{1}{2}\overline{q^2}\right)$$

was calculated using the method described by Browne et al.[3] $\partial \overline{q^2}/\partial x$ was obtained by using Eqs. (1) and (2) and the self-preserving form $\overline{q^2} = U_o^2 h(\eta)$, which gives

$$\frac{\partial \overline{q^2}}{\partial x} = U_o^2\left(\frac{dh}{d\eta}\right)\left(-\frac{\eta}{L}\right)\left(\frac{dL}{dx}\right) + 2hU_o\left(\frac{dU_o}{dx}\right)$$

Here, $dh/d\eta$ was obtained from the numerical differentiation of $\overline{q^2}/U_o^2$. This differentiation was also carried out in a manner similar to the one used for $\overline{q^2}$ diffusion. The calculated advection term is presented in nondimensional form in Fig. 4 and compared with the advection of Browne et al.[3] Both of the terms are normalized with L/U_o^3 and they compare very well with each other. The good agreement of the $\overline{q^2}$ diffusion and advection terms respectively tend to a certainty of similarity of the wake flows of the present study and of Browne et al.[3]

3. *Production*

The production of turbulent kinetic energy in self-preserving wake flow is given by the relation $\overline{uv}(\partial U/\partial y)$. The velocity derivatives $\partial U/\partial x$ and $\partial U/\partial y$ were obtained from the self-preserving form of mean velocity, which is

$$(U_\infty - U)/U_o = G(\eta) = e^{-(\ln 2)\eta^2} \tag{3}$$

The shear stress per unit mass can be obtained from the x-momentum equation in the far wake from the relation

$$-\overline{uv} = \int_o^y U_\infty \frac{\partial U}{\partial x}\,dy - \nu\frac{\partial U}{\partial y} \tag{4}$$

The production term can be either calculated using Eqs. (1–4) or can be obtained directly from the experimental results. These methods of obtaining the production of turbulent kinetic energy are in agreement. These results are shown in Fig. 5 and compared with the results of Browne et al.[3] Figure 5 clearly shows that the present results collapse on to the results of Browne et al.[3] within experimental error. This, in turn, again shows the similarity of both flows.

4. *Dissipation*

The present work is mainly directed toward the measurement of dissipation with the new method introduced by Azad and Kassab.[1] Browne et al.[3] claim the perfect way of measuring dissipation in self-similar wake flow. They have also shown that all of the terms of energy budget balance to zero, and thus, they infer that their measurement of dissipation, which agrees with this assessment, has been correctly made. We have shown that $\overline{q^2}$ diffusion, advection, and production terms of the present study agree with the corresponding terms of Browne et al.[3] Therefore, if the measurement made by the new method agrees with their results of dissipation, then it can be concluded that the zero-length method of measurement of dissipation is a correct one.

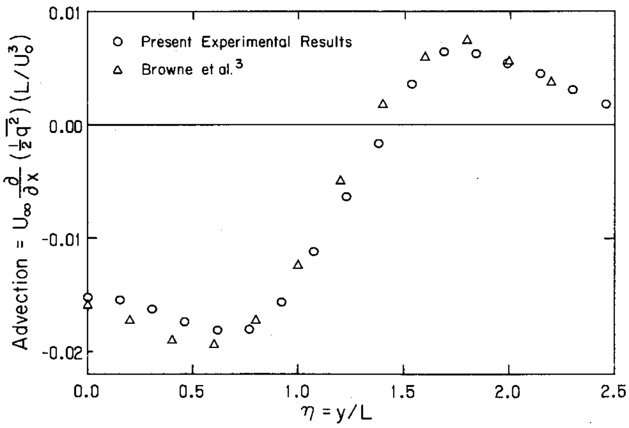

Fig. 4 Comparison of advection with Browne et al.[3]

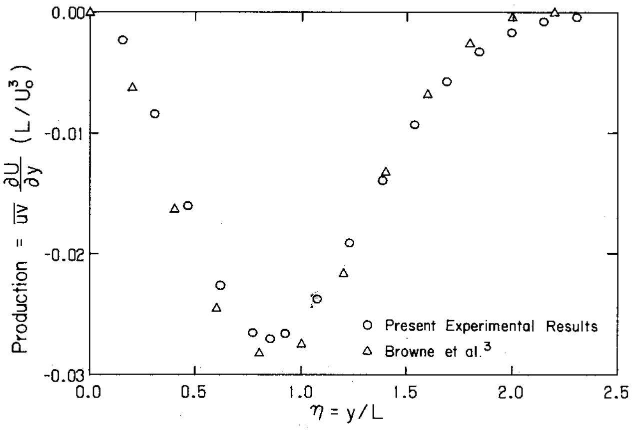

Fig. 5 Comparison of production with Browne et al.[3]

The equation used for estimating dissipation rate is the same as that used for the local isotropy from only one component of the full turbulent energy dissipation rate, that is, $\overline{(\partial u/\partial x)^2}$. However, the present estimation of the dissipation rate is made with kinetic energy spectrum of $\overline{(\partial u/\partial x)^2}$. Direct measurement of $\overline{(\partial u/\partial x)^2}$ is not possible since one wire will be in the wake of the other due to x being the flow direction unless Taylor's hypothesis is used to get $\overline{(\partial u/\partial x)^2}$. But experimentally, differentiation needs calibration and comparison with a standard to obtain a reliable value. On the other hand, the integral spectrum is equated to $\overline{u^2}$, and it is known that the measurement of $\overline{u^2}$ is most reliable. Thus, the spectral method is inherently consistent experimentally and, hence, we have adopted it.

The new method is briefly outlined in the following. In the present investigation, five different wire lengths were used to measure the $\overline{u^2}$ spectra at different locations across the wake. These spectra were used to obtain the dissipation by use of the equation

$$\varepsilon = 15\nu\int_o^\infty k_1^2 E_1(k_1)dk_1 = 15\nu\overline{\left(\frac{\partial u}{\partial x}\right)^2} \tag{5}$$

where $E_1(k_1)$ is the one-dimensional energy density defined by

$$\int_o^\infty E_1(k_1)dk_1 = \overline{u^2}$$

k_1 is the wave number in the x direction, and ν is the kinematic viscosity of the fluid. These dissipation values were plotted against the sensitive wire length and extrapolated to zero wire length. The value at zero wire length was taken as the true value of dissipation at that location. This might give the impression that the present flow is assumed to be isotropic. In fact, the isotropic relation (5) and Taylor's hypothesis are used only as a means to calculate the dissipation for a particular wire sampling the dissipation corresponding to the wire length through these relations. Thus, the dissipation in nonisotropic flows can also be measured through this method. This procedure for the location at the wake centerline is illustrated in Fig. 6.

According to one of the referees, it makes more physical sense to extrapolate the measurements in Fig. 6 not by a straight line but by a parabola that has a zero slope at the origin. The parabolic fit gives lower values of dissipation. Also, with these values of dissipation, the pressure diffusion term, which is calculated as the closing term of the energy budget, does not satisfy the integral constraint given in the following subsection. Thus, we cannot obtain true values of dissipation with a par-

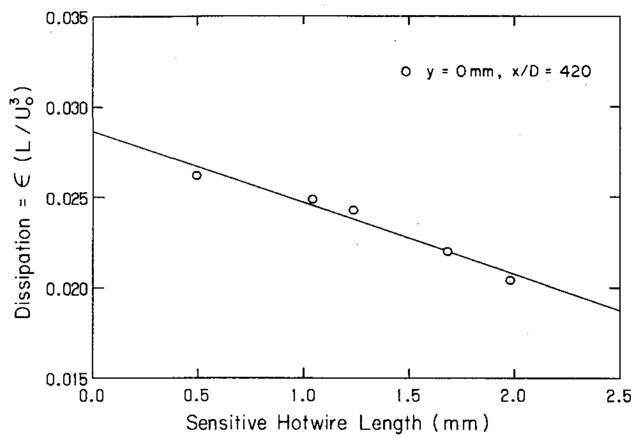

Fig. 6 Example of the zero-length method of dissipation measurement.

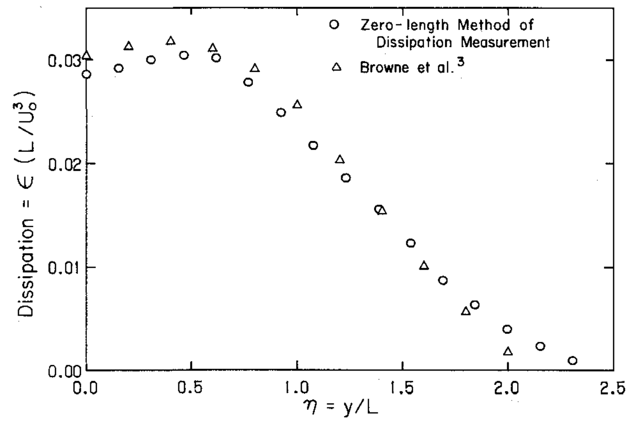

Fig. 7 Comparison of the zero-length method of dissipation measurement with the measurement of dissipation by Browne et al.[3]

abolic fit. Furthermore, at all of the y stations of measurement, the coefficient of regression for the linear fit is on an average about -0.98, which proves that it is an excellent fit for the data points. Also, according to Townsend,[11] dissipation rate is given by

$$\varepsilon = \frac{A(\overline{u^2})^{3/2}}{l_o}$$

This formula has been thoroughly investigated in low Reynolds number boundary layer by Azad and Kassab.[12] Each sensitive length of the hot wire is, in fact, proportional to this integral length l_o. Thus, the dissipation measured by them must lie on a straight line, as demonstrated in Fig. 6 as well as in Azad and Kassab.[1] The results obtained by the present method across the wake are shown in Fig. 7, where the results of Browne et al.[3] are also given. This figure clearly demonstrates the agreement between these two results. Hence, it can be concluded that the zero-length method of obtaining dissipation gives the true dissipation of turbulent kinetic energy.

5. Discussion of Energy Budget

The average turbulent kinetic energy budget can be written (see Townsend[8] and Browne et al.[3]) as

$$\underbrace{U_\infty \frac{\partial}{\partial x}\left(\frac{1}{2}\overline{q^2}\right)}_{\text{advection}} + \underbrace{\overline{uv}\frac{\partial U}{\partial y}}_{\text{production}} + \underbrace{\frac{\partial}{\partial y}\left(\frac{1}{2}\overline{vq^2}\right)}_{\overline{q^2}\text{ diffusion}} + \underbrace{\frac{\partial}{\partial y}\left(\frac{\overline{pv}}{\rho}\right)}_{\text{pressure diffusion}} + \underbrace{\varepsilon}_{\text{dissipation}} = 0 \qquad (6)$$

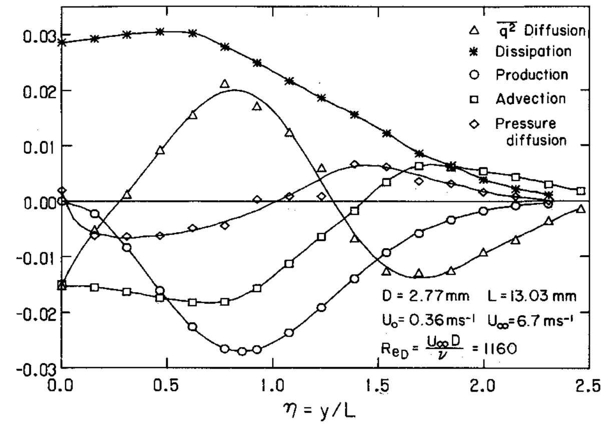

Fig. 8 Measured budget of average turbulent kinetic energy. All values normalized by L/U_o^3.

Since the four major terms, i.e., advection, production, $\overline{q^2}$ diffusion, and dissipation, are obtained from the experimental measurements, the pressure diffusion term can be calculated as the closing term of the energy budget. All of the terms of the energy budget are shown in Fig. 8. Following Browne et al.,[3] the two diffusion terms should satisfy the corresponding integral constraints,

$$\int_{-\infty}^{\infty} \frac{\partial}{\partial y}(\overline{vq^2})\, dy = 0$$

$$\int_{-\infty}^{\infty} \frac{\partial}{\partial y}\left(\frac{\overline{pv}}{\rho}\right) dy = 0$$

provided there is symmetry with respect to $\eta = 0$. In the case of Browne et al.,[3] these constraints are satisfied within 2 and 8%, respectively. In the present case, the corresponding values are 2 and 4%. These facts independently support the contention that the measurement of dissipation by the zero-length method in the present case is correct.

IV. Conclusions

The zero-length method of dissipation measurement in a turbulent wake agrees with the best measurement of dissipation available due to Browne et al.[3] in a similar flow. This method is also corroborated independently by the closure of the turbulent kinetic energy budget.

The zero-length method of obtaining dissipation has been proved to be successful in boundary-layer flow, fully developed pipe flow, and diffuser flow with high-intensity turbulence (see Azad and Kassab[1]). In addition, it has been proved to be successful in wake flow. In consequence of this information, it can be concluded that the zero-length method of obtaining dissipation is a universal method.

Acknowledgments

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